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Single-particle analog to the fractional quantum Hall effect

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Abstract

We study the spectrum of states, in a plane perpendicular to a uniform magnetic field, for an electron restricted to the lowest Landau level in the presence of randomly distributed, repulsively correlated electric impurities. The lowest energy band in this spectrum would be the lowest Landau level if there were an effective magnetic flux density downshifted by an integer multiple of the impurity density. The downshift is precisely half that for the corresponding electron density in the composite-fermion picture of the fractional quantum Hall regime. Although the work is numerical, the striking result confirms heuristic arguments based on simple adiabatic calculations. This raises a possibility that further development of adiabatic methods might allow deduction of composite-fermion theory for the fractional quantum Hall effect, starting with spin-up electrons restricted to the lowest Landau level and interacting by Coulomb forces.

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1. Introduction

Following the experimental discovery of the fractional quantum Hall effect (FQHE), Laughlin [1] gave an explanation, providing an ansatz for the many-body wavefunctions relevant to the simple FQHE fractions. In a further advance Jain [2–5] introduced the 'composite-fermion' (CF) description to account for general fractions. The CF description works well even for the compressible regime where the ratio of electron density to magnetic flux density is around $\nu = \frac{1}{2}$.[3] The evidence justifying these approaches comes from experiment, from

[3] Experimentally the $\nu = 1/2$ case seems simpler than the most detailed theory, entailing significant residual interactions, would suggest [6]. A really exotic behavior, possibly including quasiparticles with 'non-Abelian' statistics, has been suggested for $\nu = 5/2$ [7].

exact numerical computations for small systems, and from self-consistent computations for larger systems.

These successes leave little doubt that there is a theory valid in quite a wide range for the FQHE, certainly in the domain with fermionic quasiparticles. It is tempting to ask for even more: a way of deducing the FQHE from fundamental electromagnetic interactions. One may assume that only electrons in the lowest Landau level need to be considered, very nearly reducing a problem in two space dimensions to a problem in one space dimension. Thus, this might be one of those rare cases where rigorous deduction of the phenomena from the (very simple) fundamental dynamics is within reach. In practice, even though experimental and numerical evidence may give a causal link great conviction, rigorous proofs are quite rare. That is one reason why experiment has been essential to the progress of physics, and also why successful predictions from 'fundamental' to 'less fundamental' physics are uncommon.

In this paper we attempt to explore the question of how one might generate such a deductive link for this particular case, by discussing some very simple adiabatic processes, and with these as motivation, developing numerical evidence that a one-electron analog problem exhibits features remarkably like those in FQHE as described by CF theory. This striking result, the main outcome of our work, encourages us to speculate that there could be a pathway toward deduction of CF theory by analytic calculations, using the adiabatic approximation for many electrons in the lowest Landau level interacting with each other through Coulomb potentials.

The starting point in a deductive approach should be a two-dimensional gas of electrons whose screened electric interaction gives a short-range repulsion, with a magnetic field in the direction normal to the plane of electron motion so strong that only the lowest Landau level for one orientation of electron spin needs to be considered (meaning that the splitting between Landau levels is much larger than the repulsive Coulomb energies).[4]

In our numerical work we make a drastic additional simplification, by treating all electrons except one as fixed in position, and consider the resulting one-body problem of an electron moving in a magnetic field under the influence of an array of electric impurities mimicking the effect of all the other electrons. The mean density of the electrons, and therefore of the impurities, is

$$\rho \equiv \nu B, \tag{1}$$

where B is the flux density, i.e., the magnetic field strength measured in units of an Aharonov–Bohm quantum of magnetic flux. Because these electrons have strong repulsive correlations, we assume such correlations among the static impurities also. This assumption turns out to be a key feature of our model.

Studying a system with tens to thousands of impurities spread throughout a region of uniform magnetic field, we find that the one-body spectrum shows a characteristic concentration in a lowest band. We treat this band as if it were a lowest Landau level (still perturbed by residual interactions with the impurities). Then, the density of states in the band suggests an effective magnetic field downshifted from the applied field by an integer number n multiplying the density of impurities. The band is cleanly separated from one or

[4] Haldane [8] devised short-range repulsive 'pseudopotentials' from which follow as exact solutions the ground-state wavefunctions introduced by Laughlin [1] to describe the simple fractions $\nu = 1/(2n + 1)$. This represents a deduction, but from an 'artificial' starting point, in the sense that the potential—rather than being obtained from standard electrical interactions—is tailored to produce the desired ground state. That is a significant achievement, because it was not obvious that simple pseudopotentials could be found at all. Still, although the Laughlin states thus are deduced, those states are known from numerical studies not to be quite the exact configurations found in FQHE. Furthermore, the approach does not help to obtain complex fractions, or quasiparticle excitations of the ground state.

more higher energy bands, each containing a fraction ν of the total number of allowed states. The average geometrical density of states, summed over all the bands, is equal to the actual density of flux quanta, as must be true for any particular Landau level in the actual magnetic field.

In Jain's CF model, a field shift specified by an even integer q is found, where the value of q increases as the impurity density decreases. We find the remarkable result for densities in the range from $\nu \approx \frac{1}{3}$ to $\nu \approx \frac{1}{8}$, $q = 2n$, which means that in this technical sense our one-body problem accounts for exactly half the physics in the fractional quantum Hall regime.

The crucial idea motivating our study is that an electron in the presence of electric impurities and a strong magnetic field feels an induced or effective magnetic field. That is, the electric repulsion is transmuted into magnetic torque. To understand the basis for this idea, consider a two-dimensional Aharonov atom [9] in a uniform, perpendicular magnetic field. Let the attractive potential binding the electron to the (uncharged) 'nucleus' constrain the electron to occupy only degenerate states centered on the nucleus with azimuthal quantum numbers $m = 0$ or $m = 1$ of the lowest Landau level. The degeneracy will be broken by interactions of the atom with electric impurities. Imagine that the nucleus is allowed to move toward a concentrated charge at point $\mathbf{r}$. For large separations, the $m = 0$ state of the atom will give the least overlap of the electron and charge, and thus the lowest energy. However, at zero separation the $m = 1$ state of the atom will give the lowest energy.

Because there is mixing between the two states (by the electric field, which breaks azimuthal symmetry), the atom will shift from the $m = 0$ to the $m = 1$ state in the course of an adiabatic journey from large separation to zero separation between the nucleus and the fixed charge. In the limit of weak mixing, the transition from $m = 0$ to $m = 1$ takes place on a circle specified by a precise separation R between the charge and the nucleus of the Aharonov atom. Therefore, when the nucleus of the atom sits at that separation, the two wavefunctions must match everywhere on the transition circle. This can be achieved provided there is a gauge transformation matching the 'atomic' wavefunctions across the transition circle, given by the phase factor $e^{i\phi}$. The result of this gauge transformation is equivalent to an additional effective magnetic flux of one quantum, opposing the uniform magnetic field. Because the mixing by the Coulomb field is not infinitely weak, the transition circle becomes a diffuse ring, in which the effective flux is distributed. If the nucleus moves slowly in a closed curve, the effective or induced magnetic flux enclosed is an example of Berry's 'geometric magnetism' [10]. Clearly this effect could not occur if the given uniform magnetic field were not present, because there would be no way for the Coulomb repulsion to 'choose' the sign of the effective magnetic field.

The fictitious Aharonov atom suggests a more realistic example. Let us choose as our slow variable, in place of the heavy nucleus, the guiding-center coordinate for the electron, allowing in principle the full (degenerate) set of azimuthal m values with respect to this center. This would specify a complete set of states for any location of the guiding center, but the requirement of minimum energy in the presence of electric impurities should remove the consequent redundancy.

Now imagine some density of distributed electric impurities. If the density is low enough, then when the guiding center is exactly in a space between impurities, as far as possible from each of the nearest neighbors, it is energetically favorable to have $m = 0$. On the other hand, if the guiding center coincides with one of the impurities, a higher m value minimizes the energy, by keeping the electron away from the central impurity. The new m value cannot be too high, however, to avoid overlap with other impurities. Thus, the qualitative expectation is that the optimum m value for this position will increase as the impurity density decreases. The consequence will be an effective magnetic field, always of one sign, distributed throughout

the plane, and on average an integer multiple of the impurity density, with the multiple being larger as the density becomes smaller.

We shall see that such behavior indeed occurs for the lowest-energy part of the spectrum in the presence of randomly distributed, repulsively correlated impurities, little influenced by details of either the impurities or their distribution. Interestingly, the same thing does not happen for a regular lattice, most likely because some higher m values then can be degenerate with the two specified in the above discussion.

Our approach superficially resembles earlier models for genuine impurities, related to the integer quantum Hall effect. The main differences between this analysis and those studies [11–18] are (a) impurity density of less than one per flux quantum for our analysis, rather than five or more per flux quantum for the earlier studies and (b) our identification of the impurities as other electrons, distributed with repulsive correlations, as opposed to uncorrelated lattice and surface irregularities and atomic impurities.

2. Methods

To describe a single electron moving in two dimensions in a magnetic field, we use the Hamiltonian

$$H = \tfrac{1}{2}[\vec{\sigma} \cdot (\vec{p} - e\vec{A})]^2, \tag{2}$$

in units where $\hbar = c = m = 1$ and with $\vec{\sigma}$ the Pauli spin matrices. This Hamiltonian includes the interaction of the electron's spin with the magnetic field, and implies that the lowest Landau level states are spin polarized, with energy $E = 0$. We choose the cylindrical gauge and assume that the magnetic field B is strong enough that we may consider only the lowest Landau level and neglect mixing with higher levels. In keeping with the cylindrically symmetric gauge, we choose our basis states to be concentric orthonormal eigenfunctions of the form

$$\phi_m(z) = \frac{1}{\sqrt{\pi m!}} z^m \, \mathrm{e}^{-zz^*/2} \tag{3}$$

where $z = x + \mathrm{i}y = r\mathrm{e}^{\mathrm{i}\theta}$ has been expressed in units of the magnetic length

$$\ell_B = \sqrt{2/eB}. \tag{4}$$

The magnetic length is both the root-mean-square radius of the $m = 0$ basis state and the radius of the area through which one quantum of magnetic flux, $\Phi_o = 2\pi/e$, passes. The total number of flux quanta passing through the system is then given by the squared radius of the system. That radius we take as the root-mean-square radius of the largest-m basis state. Consistently with other numerical FQHE and QHE studies, e.g., [1], we construct our potential out of sums of impurities where each impurity is a pair of concentric Gaussians,

$$V_n(z) = V_0 \left[\frac{1}{\alpha^2} \, \mathrm{e}^{-|z-z_n|^2/\alpha^2} - \frac{1}{\alpha^2\beta^2} \, \mathrm{e}^{-|z-z_n|^2/\alpha^2\beta^2} \right]. \tag{5}$$

The paired Gaussians give a system with zero average electrical potential and therefore a total energy $E = 0$, and approximate a repulsive charge screened by a compensating cloud (of course all in the presence of a uniform positive background charge representing the ions of the medium).

As a starting point, we set $\beta^2 = 2$ so that the area computed from the root-mean-square radius of the negative shell would be twice that of the positive core, and then choose α so that plots of the density of states, participation ratio and root-mean-square radius as a function of energy are approximately symmetric with respect to energy when there is one impurity

for every two flux quanta. We vary both α and β to test the robustness of the model. In all cases, the widths of the Gaussians scale inversely with the square root of the impurity density, as we expect increased impurity density to increase localization of the individual impurities to minimize their overlap and hence their interaction energy. The potentials are constructed so that their space integrals are unchanged by this scaling. The impurities are randomly distributed or randomly distributed with a minimum separation between adjacent impurity centers (hard-shell constraint) or placed on a hexagonal lattice. The centers of the impurities are required to lie within the disc, but their shape is unaffected by the disc boundary. The impurity density ν is the number of impurities per flux quantum.

3. Results

We consider energy spectra and compactness, $\mathcal{C}$, of states. The latter is defined by

$$\mathcal{C} = \sqrt{P}/R_{\mathrm{rms}}. \tag{6}$$

The compactness measure uses the participation ratio, which measures the area of a state [19–21],

$$P_\alpha = \frac{(\int \mathrm{d}^2 r |\Psi_\alpha|^2)^2}{\int \mathrm{d}^2 r |\Psi_\alpha|^4} = \left(\int \mathrm{d}^2 r |\Psi_\alpha|^4 \right)^{-1}, \tag{7}$$

and the root-mean-square radius, which measures the width of the state,

$$R_{\mathrm{rms}}^{(\alpha)} = \sqrt{\int \mathrm{d}^2 r \, r^2 |\Psi_\alpha|^2}. \tag{8}$$

Compactness measures the degree to which a state resembles a disc (most compact) or a ring (least compact), and has an intrinsic scale $\sqrt{2\pi}$. For a uniform disc and for densities corresponding to the $m = 0$ and $m = 1$ basis states, the ratio has exactly the value $\sqrt{2\pi}$. This is not a maximum value, as can be seen by permitting m to be a continuous rather than a discrete variable. In the limit of large m, the states become more and more 'ring like', and the value of the ratio tends toward zero.

We have examined the energy spectra for the different kinds of distributions at impurity densities between 1 and 1100 impurities per 1000 magnetic flux quanta, for α^2 between 0.05 and 0.85, and for β^2 between 1.20 and 14.14. We find that for double Gaussian impurities randomly placed with a hard-shell constraint, the energy spectrum consists of a fraction $1 - n\nu$ of the states in the lowest energy band, and n higher energy bands each containing a fraction ν of the states, as illustrated in figure 1. For the values of α and β we have explored, the splitting is generally reasonably clear when 60–70% of the system area is enclosed in the hard-shell regions. Narrower Gaussians, i.e., smaller values of α and β, show the split for smaller hard shells than larger values do. Neither a random distribution nor a precise hexagonal close packing produces this type of energy spectrum. We also find, as illustrated in figure 2, that compactness as a function of energy for the lowest energy band is qualitatively similar to the single band that exists for one impurity per flux quantum.[5] Both of these observations suggest that the lowest band should be considered a filled Landau level in a reduced magnetic field. According to [5], when the filling factor is given by

$$\nu = \frac{p}{2pq + 1}, \tag{9}$$

[5] This is an interesting reversal from the effect of random impurities in the *absence* of a magnetic field, namely, strong localization of electrons [22]. Of course, for a plain Landau level with no perturbations, states of high and low compactness are degenerate, so that perturbations could easily turn out to favor either degree of compactness.

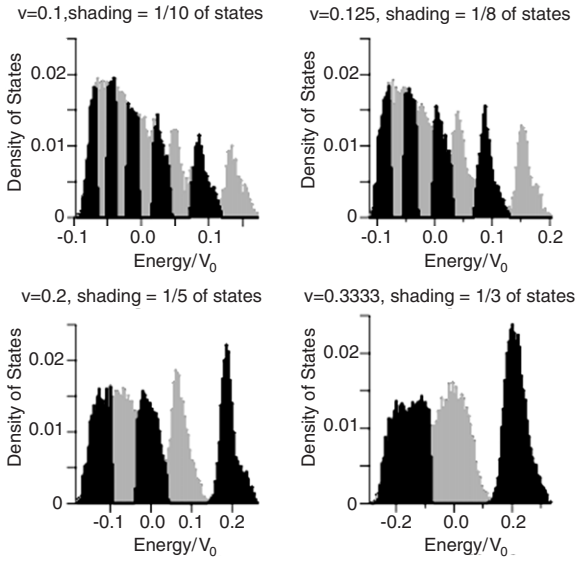

Figure 1. Summed energy histograms for ten different random distributions of impurities in the case of parameters set to $\alpha^2 = \frac{0.35}{\nu}$, $\beta^2 = 14.14$, with minimum distance between centers of impurities $0.70 \times (\text{systemsize}/\sqrt{\text{no.of impurities}})$, using matrix elements up to 201 places from the diagonal. The filling factor ν is indicated at the top of each subfigure. Shading groups states into N bands each containing a fraction $\nu = 1/N$ of the states. Note that at the higher energies two or more adjacent groups are conspicuously demarcated from each other. The lower-energy groups show less division and plausibly form a single band interpretable as a lowest Landau level in an effective magnetic field. This is most conspicuous for the largest ν value shown, $1/3$.

then the effective magnetic field is given by

$$B^* = B(1 - 2q\nu). \tag{10}$$

For ν in the interval $(1/3, 1/2)$, this means $q = 1$, for $(1/5, 1/4), q = 2$, and for $(1/7, 1/6), q = 3$.

Comparing with the results in figure 1, we see our effective field shifted down from the applied field by exactly half the amount found in the CF picture. At impurity densities falling between the discrete electron filling factors permitted in the CF picture, there is a smooth transition in the numbers of upper bands split off from the lowest band; only a limited number of bands will split off. In the vicinity of $\nu = 1/9$, the upper bands gradually lose their definition and are reabsorbed into the lowest energy band. The appearance and disappearance of banding with decreasing impurity density implies the appearance of an effective magnetic field which then disappears at still lower impurity densities. Comparison with single Gaussians and unconstrained random and crystalline distributions shows that the separation induced by the combination of the double Gaussian and the hard-shell constraint is critical to the phenomena described above. However, the separation is not critically dependent on the exact values of the parameters creating the impurity potential: α, β and the size of the hard shell. In retrospect, this is not particularly surprising. The assumptions we have made seem to constitute the closest one can get to the full electron–electron interaction in an approximation where only one electron is dynamical. It makes sense therefore that the energy banding is linked to a moderate amount of order in the system—neither too much nor too little should be present.

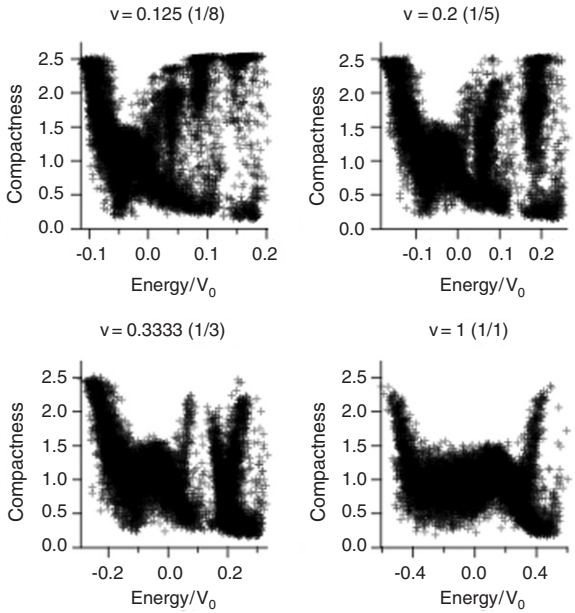

Figure 2. Combined distributions of equation (6) as a function of energy for ten different random distributions of impurities in the case of parameters set to $\alpha^2 = \frac{0.35}{\nu}$, $\beta^2 = 14.14$, minimum distance between centers of impurities is $0.70 \times$ (systemsize/$\sqrt{\text{no.ofimpurities}}$), using matrix elements up to 201 places from the diagonal and cross-terms in the participation function no more than 100 apart. The filling factor, ν, is indicated at the top of each subfigure.

4. Reflections and outlook

We have studied a model of the FQHE based on interaction of a single electron with a set of randomly distributed, repulsively correlated electric impurities (intended to mimic the influence of the electron's interaction with other electrons), finding that an effective magnetic field is induced by interaction with the impurities. This effective field downshifts the applied field by precisely half the amount known in CF theory. Of special importance is the fact that a key aspect of the model, separation of impurities, is consistent with the known behavior of electrons in the FQHE, while the particular parameter values used to achieve the separation are not critical. The fact that the induced magnetic field is not dependent on precise details of our 'frozen-electron' model suggests to us that it captures a significant aspect of FQHE physics.

These numerical results may be viewed as a vindication of the qualitative conclusions drawn earlier in the paper from considering the adiabatic approximation for motion of the guiding-center coordinate of our single electron moving among electric impurities. A natural question that might occur to a reader is this: In CF theory one finds a collection of quasi-Landau levels in the effective magnetic field. In our calculation only the lowest level appears. Why not more? The answer is quite trivial. Because the lowest level for all our examples takes up more than half the states in the lowest level for the actual magnetic field, there are not enough states available to constitute even one more quasi-Landau level. If, on the other hand, all the electrons were free to move, even though each would be restricted to the lowest Landau level, the total number of states would be so great that a large number of quasi-Landau levels

could be accommodated. In this respect also, with all its constraints our primitive calculation seems to have come as far as possible toward reproducing the FQHE.

The strongest constraint in our model was the restriction to just one dynamical electron, clearly losing much of the electron–electron dynamics needed for the FQHE. This means that we gave up any representation of Fermi–Dirac statistics, crucial to a complete theory. Partially relaxing the one-particle constraint, by considering simultaneously motion of each of a pair of electrons (in the spirit of Cooper's consideration [23] of an electron pair in the background of an electron Fermi sea as a way of understanding superconductivity), might perhaps give the full reduction of the magnetic field seen in CF theory, with wavefunction antisymmetrization selecting odd-denominator rather than even-denominator filling factors.

Thus, along the lines of our earlier discussion about the Aharonov atom and the adiabatic motion of a single guiding-center coordinate, one could take another step by considering two dynamical electrons, treating two guiding-center coordinates as adiabatic variables. To make the problem well defined, imagine motions of the guiding centers within a disc that is a potential 'hole', surrounded by a positive potential plateau extending to spatial infinity (statically representing the other electrons). Imagine that the two guiding centers initially are located midway in opposite directions along a diameter of the disc, assumed to have an area somewhere between four and six times the area corresponding to a single quantum of flux. Then, the minimum energy is obtained when each electron is in an $m = 0$ state with respect to its center.

Now imagine that both guiding centers are brought slowly to the center of the disc. At that point, the minimum energy is obtained when one of the electrons still is in an $m = 0$ state, but the other is in an $m = 2$ state, as far as possible from the inner electron without penetrating appreciably into the repulsive plateau. Because of the symmetry of the $m = 2$ and $m = 0$ wavefunctions, the adiabatic dynamics yields a symmetric combination of the two allocations, first electron with $m = 0$ and second with $m = 2$, and vice versa. Because of the Fermi–Dirac statistics of the electrons, this means that the wavefunction in the guiding-center coordinates vanishes linearly at the coincidence point. Thus one obtains naturally a description in which two units of induced magnetic flux are associated with the relative guiding-center coordinate, and these coordinates are fermionic variables, just as in CF theory. The induced magnetic flux, just as in our earlier examples, is smoothly distributed, and in general there are not multiple coincident zeros in the two-electron wavefunction—a step toward greater realism than found in the Laughlin ansatz.

What would be needed, and what could be hoped, for an eventual, rigorous program along the lines sketched here? First, the full program would entail an adiabatic approximation for simultaneous motion of the guiding-center coordinates of all the electrons. This seems a formidable task, but conceivably there could turn out to be general methods which would make it feasible. One objection to the adiabatic approach is that in most circumstances this approximation is merely an asymptotic one [10], hardly usable as a framework for rigorous deduction of anything. However, it seems to us possible that for the ground state of the $\mathcal{N}$-electron system the adiabatic approximation could be exact, at least in the limit $\mathcal{N} \to \infty$. In other words, simultaneous adiabatic motion of all the guiding-center coordinates could be describable by an exact expression, rather than an asymptotic series. The key point here is that if there were such a form arising from adiabatic analysis, it would perforce yield a ground state separated from others by a finite energy gap. Thus, the fact that without the Coulomb interactions one has a clearly gapless system is not necessarily relevant.

If it worked, the suggested adiabatic approach would have an extra advantage. In Jain's general treatment, for all except the simple fractions he obtains wavefunctions not entirely restricted to the lowest Landau level, and must make a projection to go back to that level. The

projection makes only a small difference because the higher-level admixture is small, but in the approach suggested here this would be unnecessary because from the beginning only the lowest Landau level in the actual external field appears.

The success found here using numerical analysis for a highly simplified version of the true problem seems to us encouraging for further study of the adiabatic approximation by analytic means. However, one need not agree with this speculation to be impressed by the way that the one-electron model produces an induced or effective magnetic field whose dependence on impurity density tracks so closely that of the induced field in composite-fermion theory, matching precisely once one notes an overall factor of $1/2$.

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